

## CONTROLLING FLEXIBLE STRUCTURES WITH SECOND ORDER ACTUATOR DYNAMICS

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### ABSTRACT

This paper examines the control of flexible structures for those systems with actuators that are modeled by second order dynamics. Two modeling approaches are investigated. First a stability and performance analysis is performed using a low order finite dimensional model of the structure. Secondly, a continuum model of the flexible structure to be controlled, coupled with lumped parameter second order dynamic models of the actuators performing the control is used. This model is appropriate in the modeling of the control of a flexible panel by proof-mass actuators as well as other beam, plate and shell like structural members. The model is verified with experimental measurements.

### INTRODUCTION

This paper presents a study of active vibration suppression in flexible structure using actuators with second order dynamics. First, a low order model of the structure is used to investigate stability properties. It is then shown that the common practice of maintaining stability by using relative velocity feedback (i. e. the difference between the structure's velocity and the actuator mass velocity) does not necessarily lead to the best closed loop performance.

In addition to a finite dimensional analysis of the effects of actuator dynamics on active vibration suppression, an infinite dimensional model is suggested. This model proposes adapting an approach presented by previous authors on combined dynamical systems. This approach is adapted here to include a control law acting between a (finite dimensional) second order actuator and a structure defined by a second order in time partial differential equation. The specific case of a beam modeled as an Euler-Bernoulli equation with both internal (Kelvin-Voight) and external (air) damping included and actuator is presented. Modal equations are presented in terms of the Green's function without actuator dynamics. The case of velocity feedback of the structure at the point of attachment on the beam is derived. The open loop equation with point dynamics are then verified experimentally. A short list of acronyms used in this paper follows in the appendix.

### SDOF/PMA

The linear proof mass actuator (PMA) is a solenoid-like device<sup>(3)</sup>. Current flowing in a coil of wire attached to the frame of the PMA, combined with permanent magnets fixed to the proof mass produces an electromagnetic force that accelerates the proof mass. Furthermore, this

electromagnetic force produces a reaction force on the PMA frame that can be used for control law actuation. By regulating the current supplied to the coil one can control the force produced by the actuator. A linear variable differential transformer (LVDT) is used to measure  $\eta$ , the proof mass's position relative to the PMA frame. This signal can be differentiated to give  $\dot{\eta}$ , the proof mass's velocity relative to the frame. An accelerometer attached to the PMA gives the structural acceleration, which is integrated to give its velocity,  $\dot{x}_s$ . These three measurements provide three local feedback paths that can be used for output feedback control. The combination of the proof mass and these feedback paths can be modeled as a second order system as shown in fig. 1.

In fig. 1 the structure to be controlled is represented by a damped single degree of freedom system,  $(M_s, K_s, C_s)$ , the actuator is also modelled by a single degree of freedom system with an accompanying force generator. The spring stiffness,  $k_a$ , represents the servo stiffness of the actuator. The damper,  $c_a$ , represents the friction inside the actuator. A force generator,  $f_g$ , is used to show the use of either velocity feedback path. The dead mass,  $m_d$ , associated with the actuator is fixed to  $M_s$ . The equations of motion for this system are

$$\begin{bmatrix} M_c & m_p \\ m_p & m_p \end{bmatrix} \ddot{\mathbf{X}} + \begin{bmatrix} C_s & 0 \\ 0 & c_a \end{bmatrix} \dot{\mathbf{X}} + \begin{bmatrix} K_s & 0 \\ 0 & k_a \end{bmatrix} \mathbf{X} = \begin{bmatrix} 0 \\ f_g \end{bmatrix} \quad (1)$$

$$\mathbf{X} = \begin{bmatrix} x_s & \eta \end{bmatrix}^T, \quad M_c = M_s + m_d$$

In order for any control formulation to be useful it must be stable. Therefore, the effects on stability of each of the feedback paths must be examined.

The relative position feedback gain,  $k_a$ , must be positive<sup>(4)</sup> for the stiffness matrix to be positive definite. If  $k_a$  were zero the actuator would have an uncontrolled rigid body mode. Therefore, a first requirement for stability is that  $k_a$  be positive.

With relative velocity feedback, i.e.  $f_g = -c_g \dot{\eta}$ , the damping matrix becomes

$$\mathbf{C} = \begin{bmatrix} C_s & 0 \\ 0 & c_a + c_g \end{bmatrix} \quad (2)$$

Which is symmetric and positive definite for all values of  $c_g > 0$ , making this a stable control law. Yet, the term  $c_a + c_g$  in the damping matrix casts this type of control into the same formulation as a traditional vibration absorber problem<sup>(1,10)</sup>.

If the control force is made proportional to the structure's velocity, i.e.  $f_g = -g \dot{x}_s$ , the damping matrix becomes

$$\mathbf{C} = \begin{bmatrix} C_s & 0 \\ g & c_a \end{bmatrix} \quad (3)$$

Which is asymmetric, such that the system

$$\mathbf{M} \ddot{\mathbf{X}} + \mathbf{C} \dot{\mathbf{X}} + \mathbf{K} \mathbf{X} = 0 \quad (4)$$

becomes potentially unstable as  $g$  increases. Therefore, the question arises: why use  $f_g = -g\dot{x}_s$  if this type of control can become unstable? If one were designing a colocated velocity control law that ignored the actuator dynamics, this is exactly what one would do<sup>(9)</sup>. Our purpose here is to determine whether using non-colocated velocity feedback will provide performance benefits over the use of only colocated, while including the actuator dynamics in the analysis.

Finally, a combination of both relative velocity feedback and direct velocity feedback is considered rather than each path individually. That is  $f_g = -g\dot{x}_s - c_g\dot{\eta}$  which produces no new stability problems, yet provides for the interaction of both feedback paths. In other words, the use of the colocated feedback can be used to stabilize the non-colocated feedback. Therefore, the problem becomes that of finding an optimal choice for  $k_a$ ,  $c_g$ , and  $g$ . This problem will be formulated as a linear quadratic regulator and solved as a parameter optimization.

The cost function is chosen to be the infinite time integral of the square of the structure's position,

$$J = \int_0^{\infty} \dot{x}_s^2 dt = \int_0^{\infty} \mathbf{q}^T \mathbf{Q} \mathbf{q} dt \quad (5)$$

$$\mathbf{q} = [x_s \quad \eta \quad \dot{x}_s \quad \dot{\eta}]^T$$

$$Q_{ij} = \begin{cases} 1 & i=j=1 \\ 0 & \text{otherwise} \end{cases}$$

The equations of motion are written in state space form

$$\dot{\mathbf{q}} = \mathbf{A} \mathbf{q} \quad (6)$$

$$\mathbf{A} = \begin{bmatrix} 0_{2 \times 2} & \mathbf{I}_{2 \times 2} \\ -\mathbf{M}^{-1} \mathbf{K} & -\mathbf{M}^{-1} \mathbf{D} \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} M_s + m_p & m_p \\ m_p & m_p \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} C_s & 0 \\ g & c_a + c_g \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} K_s & 0 \\ 0 & k_p \end{bmatrix}$$

The cost function can be rewritten as

$$J = \mathbf{q}^T(0) \mathbf{P} \mathbf{q}(0) \quad (7)$$

where  $\mathbf{P}$  represents the solution of a Lyapunov equation

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} = -\mathbf{Q} \quad (8)$$

Note, that in comparison to a typical LQR optimal control problem, that is

$$J = \int_0^{\infty} \mathbf{q}^T \mathbf{Q} \mathbf{q} + \mathbf{u}^T \mathbf{R} \mathbf{u} dt \quad (9)$$

there is no penalty on the control here, i. e.  $\mathbf{R} = 0$ . In fact there is no control,  $\mathbf{u}$ , in this problem, only feedback gains which are treated as parameters. Hence, while this problem resembles an optimal control problem it is really a parameter optimization.

The cost function  $J$  is a function of both the initial conditions and  $P$ , where  $P$  is a matrix whose elements are a function of the feedback gains. To remove the dependence of  $J$  on  $q(0)$  the initial conditions are treated as a random vector, and the expected value of  $J$  is taken as

$$E(J) = E\{q^T(0)Pq(0)\} = \text{tr}(PZ^o) \quad (10)$$

$$Z^o = q(0)q^T(0) \quad (11)$$

$Z^o$  is a normalized second order moment matrix of the initial conditions. For this example it is assumed that the structure and actuator are at rest with the structure given an initial deflection and the proof mass at its equilibrium position relative to the structure. Therefore  $Z^o$  is

$$Z^o = \begin{cases} 1 & i=j=1 \\ 0 & \text{otherwise} \end{cases}$$

The final expression for the cost function is

$$J = P_{11} \quad (12)$$

A necessary condition for optimality is that

$$\left. \frac{dJ}{dF} \right|_{F^*} = 0 = \left. \frac{dP_{11}}{dF} \right|_{F^*} \quad (13)$$

where  $F$  represents the set of feedback gains. The matrix  $P$  was computed algebraically, with the aid of MACSYMA, and the required derivatives calculated analytically. With these analytical expressions the optimal feedback gains were determined numerically. A special case of this formulation is for zero structural velocity feedback,  $g=0$ , the equations simplify substantially and the results of Juang<sup>(1)</sup> are obtained.

### Example

Consider attaching a PMA to the tip of an undamped cantilevered beam, with the desire to attenuate the first bending mode of vibration of this beam. The material properties of the beam are given in Table 1<sup>(6)</sup>. The mass properties of the PMA are given in Table 2.

The first natural frequency for the beam plus dead mass calculated from a finite element analysis is 12.66 rad/sec (2.02 Hz). The equivalent bending spring stiffness is calculated to be 158.2 N/m, and the structural first modal mass plus actuator dead mass is 0.987 kg. Table 3 gives the optimal parameter settings for two conditions: 1) zero structural velocity feedback (i. e., a vibration absorber), and 2) with structural velocity feedback and the relative velocity feedback gain held to 17.5 N-s/m.

Fig. 2 shows the structural response of both systems to the same initial condition. Note that, the settling time, and maximum overshoot of system 2 is superior to that of system 1. Fig. 3 shows the response of the actuator mass for this simulation. The major disadvantage of this control law can be seen in this figure, which is the actuator of system 2 requires a greater stroke length.

There are several lessons that can be learned from controlling a single degree of freedom structure with a PMA that can be applied to the vibrational control of both multiple degree of freedom and distributed parameter structures. Control laws that ignore actuator dynamics may result in closed loop instability. The use of only safe or nondestabilizing feedback paths may not yield the best

performance. Furthermore, using only relative position and velocity feedback results in a control law that is no different than that of a traditional vibration absorber. This type of design tends to require low feedback gains, such that the motion of the proof mass is unimpeded. Finally, better performance is achieved with structural velocity feedback combined with relative velocity feedback. In fact, a high structural feedback gain can only be tolerated in the presence of a high relative velocity feedback gain.

## MDOF/MPMA

Fig. 4 shows two proof mass actuators attached to four degree of freedom structural model. Assume that the actuators are not interconnected. That is, any measurements made by PMA one are not shared with PMA two and vice versa. In this case we have the same feedback paths as in the SDOF/PMA, case just more of them.

The relative position feedback gain,  $k_{pi}$ , must be positive for each actuator  $i$ . This requirement eliminates the uncontrolled rigid body mode of the actuator. Furthermore,  $k_{pi} > 0$  is necessary so that the stiffness matrix for this system is positive definite. Similarly, the combined relative velocity gain for each actuator,  $(c_a + c_g)_i$  must be positive. The intuitive proof for these requirements is that the addition of a damped single degree of freedom to a stable damped multiple degree of freedom structure results in a stable system.

If structural velocity feedback is used, the stability of the system can be examined using the well known Routh-Hurwitz criteria. For high order systems this test becomes difficult to implement. An alternative is to examine the system damping matrix. For multiple degree of freedom systems that can be described by the system of equations, Eq. (4), stability is guaranteed if the symmetric portion of the damping matrix is positive definite, provided that the mass and stiffness matrices are symmetric positive definite. The damping matrix  $C$  is made asymmetric by the introduction of the feedback gains  $g_i$  in some of the off diagonal terms. Any asymmetric matrix can be written as the sum of a symmetric and a skew symmetric matrix.

$$C = C_{\text{sym}} + G, \quad C_{\text{sym}} = C_{\text{sym}}^T \quad G = -G^T$$

Therefore, the stability of the MDOF/ MPMA system is guaranteed if the matrix  $C_{\text{sym}}$  is positive definite<sup>(7)</sup>. This test can be applied to the system of the previous section. The matrix  $C_{\text{sym}}$  for this system is

$$C_{\text{sym}} = \begin{bmatrix} C_s & \frac{g}{2} \\ \frac{g}{2} & c_a + c_g \end{bmatrix} \quad (14)$$

and will be positive definite if the following relations are satisfied

$$C_s > 0 \quad (15)$$

$$4 C_s (c_a + c_g) > g^2 \quad (16)$$

Determining whether high order matrix with a small number of feedback gains is positive definite can become just as tedious as the full Routh-Hurwitz test. Therefore, it is useful to simplify this result by examining the definiteness of the principal minors of  $C_{\text{sym}}$ . The matrix  $C_{\text{sym}}$  can be written as the sum of a positive definite matrix  $C_c$  and a sparse matrix of zeros and 2x2 symmetric blocks containing  $g_i$  placed along the diagonal according to the actuator placement. When this sparse block diagonal matrix is added to  $C_c$  to form  $C_{\text{sym}}$  the only minors of  $C_c$  that are affected

are the 2x2 blocks added at the position of the actuator coordinates. Therefore, only those minors changed by the addition of feedback need be checked. This test leads to the following result for each actuator<sup>(8)</sup>.

$$4(C_{si} + C_{si+1})(c_a + c_g)_i > g_i^2 \quad (17)$$

It should be recalled that these results if satisfied ensure stability, but if violated do not imply instability. Therefore, we feel that this stability criteria is of a conservative nature.

### DPS/PMA

In this section we examine a beam modelled as an Euler-Bernoulli beam with both air damping and Kelvin-Voight internal strain rate damping. A proof mass actuator is attached to the beam at some point, fig. 5. The analysis here follows that of Bergman<sup>(2)</sup>. The equations of motion for this system are:

$$\rho u_{tt} + (c_1 + c_2 I \frac{\partial^4}{\partial x^4}) u_t + EI \frac{\partial^4}{\partial x^4} u = \sum_{i=1}^r \{ [F_i(t) - f_{gi}(t) - m_{di} u_{tt}(h_i, t)] \delta(x-h_i) \} \quad (18)$$

$$m_{pi} \ddot{\eta}_i + m_{pi} u_{tt}(h_i, t) + c_{pi} \dot{\eta}_i + k_{pi} \eta_i = f_{gi}(t) \quad (19)$$

$$F_i(t) = -m_{pi} \ddot{\eta}_i - m_{pi} u_{tt}(h_i, t) + f_{gi}(t) \quad (20)$$

$$f_{gi}(t) = -c_{gi} u_t(h_i, t) \quad (21)$$

$r$  = the number of actuators

Where  $c_{gi}$  is the structural velocity feedback gain. Note that in this case only feedback of the local velocity is considered.

These equations can then be nondimensionalized according to the following definitions

$$\begin{aligned} \bar{u} &= \frac{u}{l}, & \bar{\eta} &= \frac{\eta}{l}, & \bar{\xi} &= \frac{x}{l}, & \bar{\delta}_i &= \frac{h_i}{l}, \\ \tau &= \Omega t, & \Omega^2 &= \frac{EI}{\rho l^4}, & \delta(\bar{\xi} - \bar{\delta}_i) &= l \delta(x-h_i), \\ \epsilon_1 &= \frac{c_1}{\rho \Omega}, & \epsilon_2 &= \frac{c_2 I}{\rho \Omega l^4}, & \bar{F}_i(t) &= \frac{l^2}{EI} F_i(t), \\ \mu_{di} &= \frac{m_{di}}{\rho l}, & \mu_{pi} &= \frac{m_{pi}}{\rho l}, & \bar{f}_i(t) &= \frac{l^2}{EI} f_i(t), \\ K_i &= \frac{k_{pi} l^3}{EI}, & \alpha_{oi} &= \frac{K_i}{\mu_{pi}}, & C_{gi} &= \frac{l^3 \Omega c_{gi}}{EI} \end{aligned}$$

The nondimensional equations of motion are, where the overbars have been dropped

$$u_{\tau\tau} + (\epsilon_1 + \epsilon_2 \frac{\partial^4}{\partial \xi^4}) u_\tau + \frac{\partial^4}{\partial \xi^4} u = \sum_{i=1}^r \{ [-\mu_{pi} \ddot{\eta}_i - \mu_{pi} u_{\tau\tau}(\delta_i, \tau) - \mu_{di} u_{\tau\tau}(\delta_i, \tau)] \delta(\xi - \delta_i) \} \quad (22)$$

$$\ddot{\eta}_i + u_{\tau\tau}(\delta_i, \tau) + \epsilon_i \dot{\eta}_i + \alpha_{oi} \eta_i = -\frac{1}{\mu_{pi}} C_{gi} u_\tau(\delta_i, \tau) \quad (23)$$

The solution to this problem is assumed to be of the form

$$u(\bar{\xi}, \tau) = \sum_{n=1}^{\infty} a_n(\tau) X_n(\xi) \quad (24)$$

$$\eta(\tau) = \sum_{n=1}^{\infty} a_n(\tau) A_{in} X_n(\delta_i) \quad (25)$$

where

$$A_{in} = \frac{\alpha_n^4}{\alpha_{oi}^4 - \alpha_n^4}, \quad \tilde{A}_{in} = A_{in} + 1 = \frac{\alpha_{oi}^4}{\alpha_{oi}^4 - \alpha_n^4} \quad (26)$$

and  $X_n(\xi)$  are the eigenfunctions of the undamped system.  $\alpha_{oi}^4$  represents the square of the actuator's undamped natural frequency, and  $\alpha_n$  is the eigenvalue corresponding to  $X_n(\xi)$ . For the special case of zero actuator dead mass, the temporal coefficients satisfy

$$\ddot{a}_n(\tau) + (\epsilon_1 + \epsilon_2 \alpha_n^4) \dot{a}_n(\tau) + \alpha_n^4 a_n(\tau) + \sum_{m=1}^{\infty} \left\{ C_{gi} \left( 1 - \frac{1}{\mu_{pi} A_n A_m} \right) + \epsilon \frac{\alpha_n^4 \alpha_m^4}{\alpha_o^8} - \epsilon_1 - \epsilon_2 \alpha_n^4 \right\} \mu_{pi} A_{in} A_{im} X_n(\xi) X_m(\xi) \dot{a}_n(\tau) = 0 \quad (27)$$

The eigenvalues  $\alpha_n^4$  are calculated from

$$\sum_{i=1}^r \left\{ \left[ \delta_{ij} - \mu_{pi} \frac{\alpha_{oi}^4 \alpha_n^4}{\alpha_{oi}^4 - \alpha_n^4} G(\delta_i, \delta_j; \alpha_n) \right] X_n(\delta_i) \right\} = 0 \quad (28)$$

where  $G$  is the Green's function for a clamped - free Euler-Bernoulli beam. These equations are then useful for stability analysis, control gain formulation, and experimental verification.

## EXPERIMENTAL RESULTS

A combined lumped parameter zero gain distributed parameter experiment was performed to verify Eq. (28) for the special case of  $\mu_{di} = 0$ . A clamped-free Euler-Bernoulli beam, whose material properties are given in table one, was appended with two identical passive spring-mass systems. One was placed in the middle of the beam,  $x = 1/2$ , and one at the free end,  $x = 1$ . The mass was measured to be  $49.2 \times 10^{-3}$  kg. The spring stiffness was 858 N/m.

In a test, an accelerometer was placed at the free end of the beam to measure the response of the structure. An impact hammer was used to excite the beam with hammer hits made at various points along the length the beam. The first nine natural frequencies were estimated from the experimental data using a Nyquist plot circle fitting technique. These same natural frequencies were calculated by numerical solution of Eq. (28) for the first nine modes. These results are given in table four.

Apparently, the estimated frequencies agree very well with the theoretically predicted ones. In fact, the standard deviation of the estimated frequencies to the theoretical is generally small. The error between the estimates and predictions is on the order of 1%.

## SUMMARY

Actuators that can be modeled as lumped second order systems were examined for use in vibration control of a distributed parameter system. A finite dimensional model provided insight that was extended to the distributed parameter case. It was seen that ignoring the actuator dynamics can lead to an unstable control law formulation. Secondly, the feedback paths available for output feedback control were identified and examined in terms of closed loop stability. This resulted in closed loop stability conditions for computing stable control gains. Finally, an example was given where it was seen that the use of noncolocated feedback gave better performance than solely colocated feedback.

An infinite dimensional formulation of a cantilevered beam with actuator dynamics was presented and experimentally verified. This model included both structural damping, in the form of viscous and Kelvin-Voight damping, and the actuator dynamics. It remains to complete the computational studies of the infinite dimensional model and its approximations. Initial experimental verification showed good agreement between theoretically predicted and experimentally estimated natural frequencies.

## ACKNOWLEDGEMENT

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## APPENDIX: ACRONYMS

DPS - Distributed Parameter System  
LVDT - Linear Variable Differential Transformer  
MDOF - Multiple Degree of Freedom  
MPMA - Multiple Proof Mass Actuator  
PMA - Proof Mass Actuator  
SDOF - Single Degree of Freedom

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Table 1. Beam parameters

Property	Units	Symbol	Value
Length	m	$l$	1.0
Moment of Inertia	$m^4$	$I$	$1.64 \times 10^{-9}$
Area, cross section	$m^2$	$A$	$0.597 \times 10^{-3}$
Young's Modulus	GPa	$E$	26.8
Viscous damping	$N \ s \ m^{-1}$	$c_1$	1.75
Kelvin-Voight damping	$kN \ s \ m^{-1}$	$c_2$	20.5
linear Density	$kg \ m^{-1}$	$\rho$	1.02

Table 2. Actuator mass parameters

Property	Unit	Symbol	Value
Proof mass	kg	$m_p$	0.225
Dead mass	kg	$m_d$	0.730

Table 3. Example gain settings

Gain	System 1	System 2
$K_a \ (N \ m^{-1})$	23.9	211
$g \ (N \ s \ m^{-1})$	0	77.5
$c_a + c_g \ (N \ s \ m^{-1})$	1.0	17.5
Cost $J$	0.183	0.0710

Table 4. Measured and predicted frequencies

Mode	Analytical Freq. (Hz)	Experimental Freq. (Hz)	Std dev.
1	3.49	3.48	0.127
2	17.7	17.8	0.233
3	22.4	22.6	0.0479
4	29.3	28.5	0.916
5	68.3	67.8	0.517
6	133	132	0.995
7	219	217	1.30
8	327	324	3.24
9	457	452	4.40

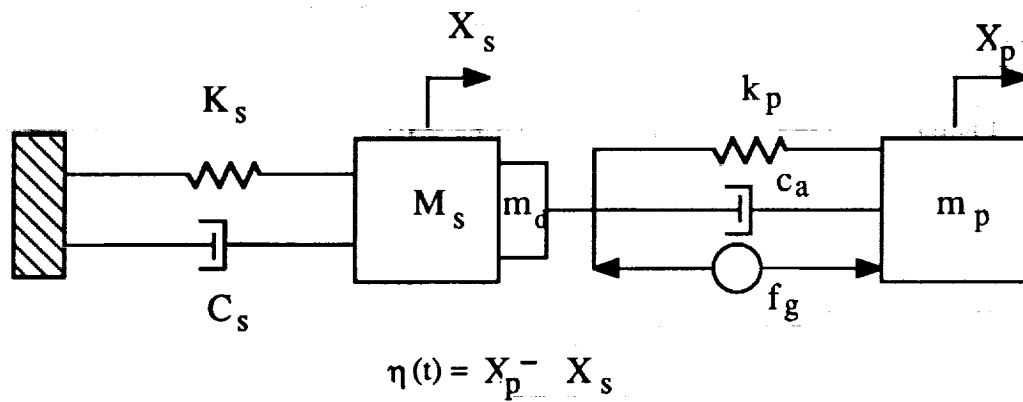


Figure 1. Single degree of freedom structure with proof mass actuator (SDOF/PMA)

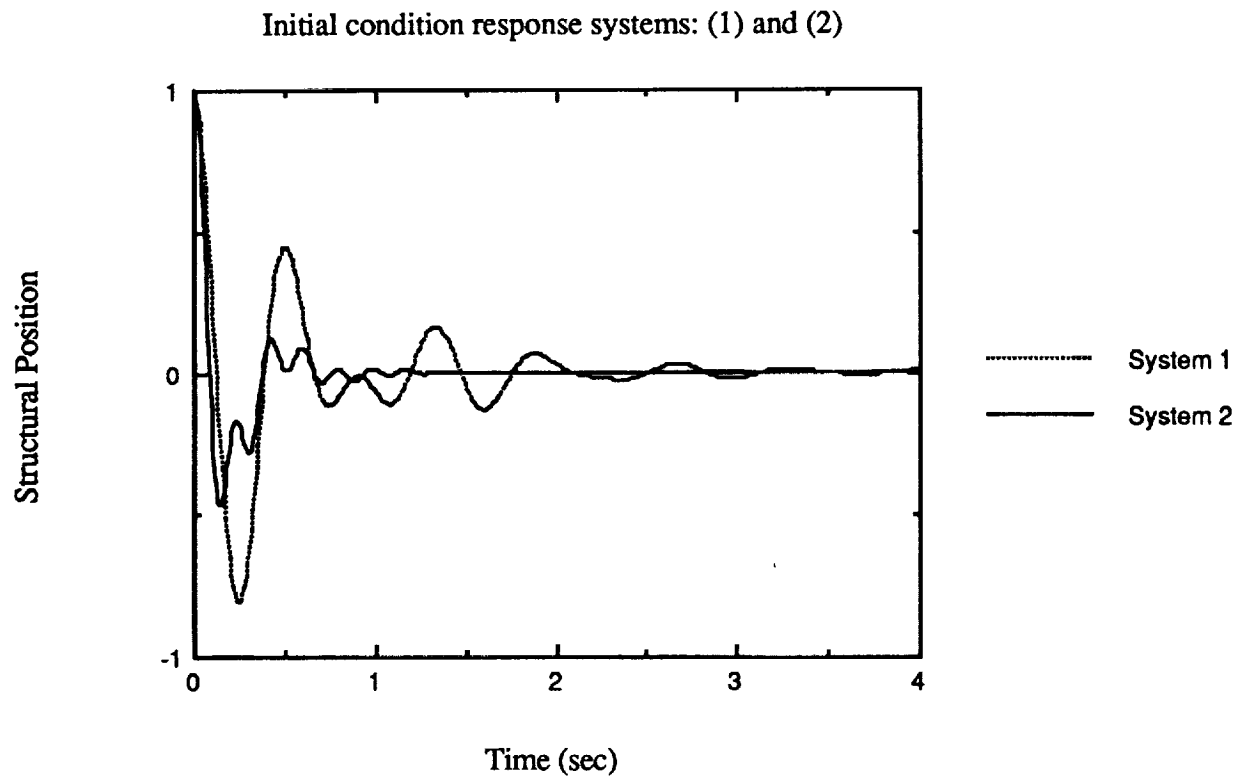


Figure 2. Structural position response: system (1) vibration absorber  
system (2) active control with velocity feedback

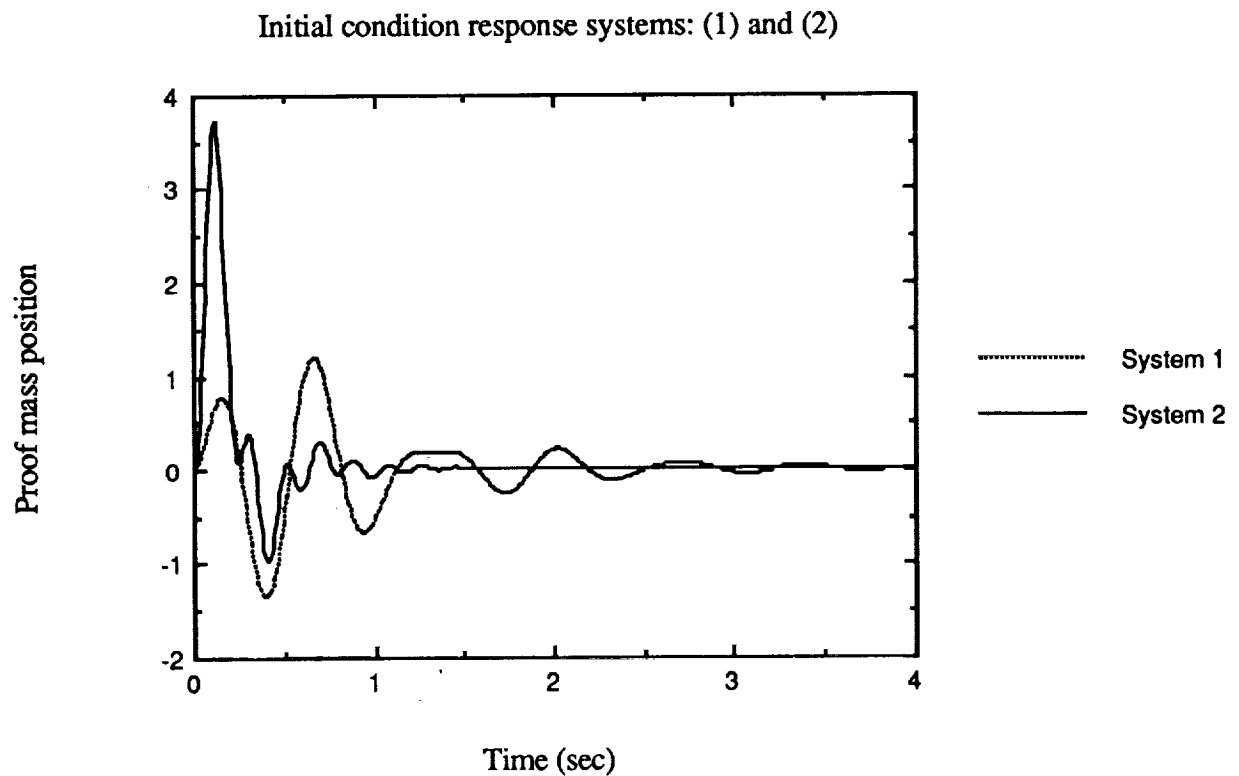


Figure 3. Actuator mass response: system (1) vibration absorber  
system (2) active control with velocity feedback

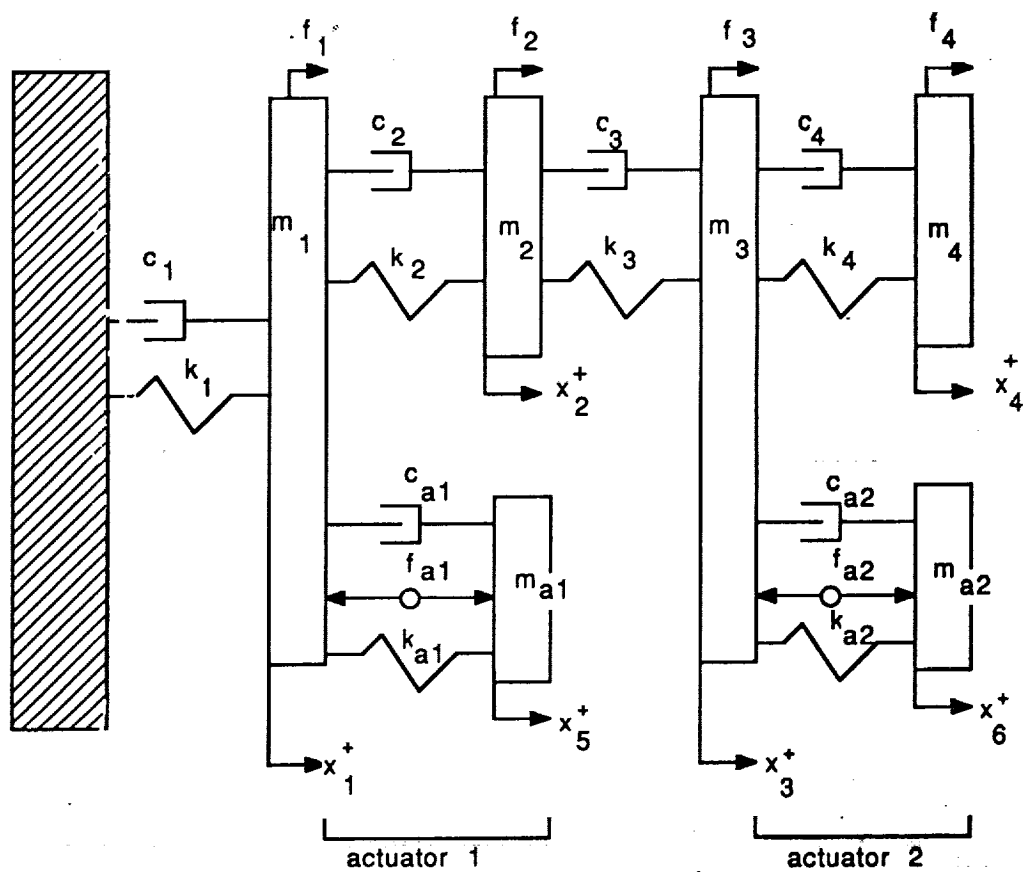


Figure 4. Multiple degree of freedom structure with multiple actuators (MDOF/MPMA)

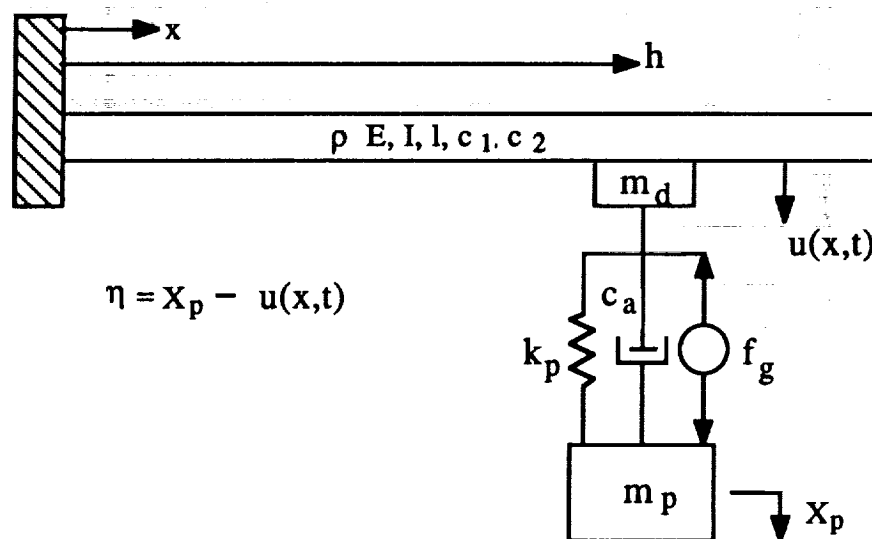


Figure 5. Euler-Bernoulli beam with proof mass actuator (DPS/PMA)